STATS 2MB3, Tutorial 4

Feb 6th , 2015

Distribution of linear combination

- $E(aX+bY+cZ)=aE(X)+bE(Y)+cE(Z)$ regardless of whether X,Y and Z are independent or not.
- $Var(aX+bY)=a^2\ Var(X)+b^2\ Var(Y)+c^2Var(Z)$ only if X, Y and Z are independent.
- $Var(aX+bY+cZ)= a^2\ Var(X)+b^2\ Var(Y) + c^2Var(Z) +abCov(X,Y)+bcCov(Y,Z)+acCov(X,Z)$ is the general form.
- $Var(X+Y)=Var(X-Y)$ if X and Y are independent.
- The linear combination of normal variables is still a normal variable.

Ex 49, Page 229

- There are 40 students in the class. The time for grading a randomly chosen exam paper is a random variable with expected value 6 min and standard deviation 6 min.
- a) If the grading times are independent and the grader begins at 6:50pm, then what is the probability it ends before 11:00pm?
- b) What is the probability that the grader finishes the work after 11:10pm?

a)

Since there are 40 students, then we set the grading times as X_1 , X_2 , …, X_{40} .

The total grading time is $T = X_1 + X_2 + \ldots + X_{40}$.

The expected value and standard deviation of T are

$\mu_T = n\mu = 40*6 = 240$, $\sigma_T = n^{1/2}\sigma = 40^{1/2} *6 = 37.95$.

There are 250 minutes between 11pm and 6:50pm.

Hence, $P(T \leq 250)$

$\approx P((T - n\mu)/ n^{1/2}\sigma \leq (250-240)/37.95)$

$= P(Z \leq 0.26) = 0.6026$.

b)

Since 11:10pm-6:50pm=260min, then

$P(T>260) \approx P((T- n\mu)/ n^{1/2}\sigma >(260-240)/37.95)$

$=P(Z>0.53)=0.2981.$

Ex 65, page 234

- When pH of a certain chemical compound is measured by randomly selected student is 5.00 with standard deviation 0.2. Let $\overline{X}$ be the average pH measured by students in morning lab and $\overline{Y}$ be the average pH measured by students in afternoon lab.
- a) If pH is a normal distributed variable and there are 25 students in each lab, compute $P(-0.1 \leq \overline{X} - \overline{Y} \leq 0.1)$.

  b) If there are 36 students in each lab, but pH is not normal distributed, calculate $P(-0.1 \leq \overline{X} - \overline{Y} \leq 0.1)$

- a) $E(\overline{X}-\overline{Y})=5-5=0,$

$$\mathrm{Var}(\overline{X}-\overline{Y})=\frac{\sigma^2}{25}+\frac{\sigma^2}{25}=\frac{2}{25}\cdot 0.2^2=0.0032,$$

$$sd(\overline{X}-\overline{Y})=\sqrt{0.0032}=0.0566.$$

Since $\dfrac{(\overline{X}-\overline{Y}-\mathrm{E}(\overline{X}-\overline{Y}))}{sd(\overline{X}-\overline{Y})}\sim N(0,1)$

then

$$\mathrm{P}(-0.1\le \overline{X}-\overline{Y}\le 0.1)=\mathrm{P}(\frac{-0.1-0}{0.0566}\le \mathrm{Z}\le \frac{0.1-0}{0.0566})=\mathrm{P}(-1.77\le \mathrm{Z}\le 1.77)$$

the probability is 0.9232

- b)
- Since 36 is sufficiently large, then we can use Central limit theorem.

$$Var(\overline{X}-\overline{Y}) = \frac{\sigma^2}{36}+\frac{\sigma^2}{36} = 0.0022$$

$$sd(\overline{X}-\overline{Y}) = \sqrt{Var(\overline{X}-\overline{Y})} = 0.0471$$

$$\mathrm{P}(-0.1 \le \overline{X}-\overline{Y} \le 0.1) \approx \mathrm{P}\left(\frac{-0.1-0}{0.0471} \le \mathrm{Z} \le \frac{0.1-0}{0.0471}\right) = \mathrm{P}(-2.12 \le \mathrm{Z} \le 2.12)$$

The probability is 0.9660.

Ex 86, page 237

- A student has a class that is supposed to end at 9:02am with standard deviation 1.5 min (normal distribution) and the starting time of next class is 9:10am with standard deviation 1 min (normal distribution). Suppose the time from one classroom to another is normally distributed with mean 6 min and standard deviation 1 min. What is the probability that the student gets the second classroom before the lecture starts?

- There are 8 minutes between 9:02am and 9:10am. Then we can set three random variables to represent the ending time of first class(X_1), the starting time of second class(X_2) and time consumed on the way(X_3). And

  $X_1 \sim N(0,1.5^2)$

  $X_2 \sim N(8, 1)$

  $X_3 \sim N(6, 1)$

We need to calculate the probability

$P(X_1 + X_3 < X_2) = P(X_1 + X_3 - X_2 < 0)$

- Since

  $X_1 + X_3 - X_2 \sim N(-2, 4.25)$,

then

$P(X_1 + X_3 - X_2 < 0) = P(Z < 2.06) = 0.9803$